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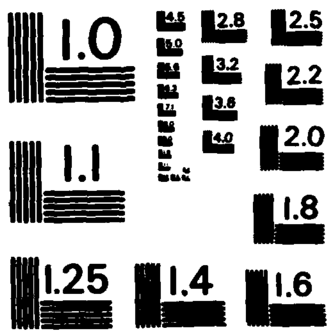

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ON THE OPTIMALITY OF DATA PROCESSORS FOR SIGNAL DETECTION OVER A CLASS OF CONTAMINATED NOISES

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ABSTRACT

We consider the effect induced on the data processor of a signal detection system when the underlying noise distribution functions are varied about their nominal values. We first consider the detection of a time varying deterministic signal in additive noise, and then extend our results to a more general situation in which the signal possesses a random amplitude. Our results characterize a class of contaminants of an arbitrary nominal distribution over which the data processor can be designed using the nominal distribution, and it is seen, for example, that such a class can contain distribution functions which can differ greatly from the nominal distribution.

I. INTRODUCTION

Consider the discrete time detection of a signal in corrupting noise. It is well known that a typical corresponding detector consists of a data processor followed by a threshold comparator. In fact, it is also well known that under a variety of fidelity criteria the data processor is characterized in terms of the relevant likelihood ratio. We note that it is often convenient in practice to employ an appropriate function of the likelihood ratio for the data processor. However, in order to present specific results about the general situation we will focus attention on the likelihood ratio itself.

An important factor which often limits the employment of the likelihood ratio within the data processor is inexact knowledge of the underlying statistical distributions. One way of resolving this difficulty is to employ a robust detector, such as the kind obtained from Huber's results (e.g. see [1]). Numerous authors have investigated various aspects of robust detection [2-6]. While robustness approaches can achieve the desired goal of desensitizing the data processors to inexact statistical knowledge, the would-be user of such detectors quickly becomes aware of their inherent limitations (for example, the extension of Huber's results to account for realistic forms of dependency and effects of nonstationarity, the question of the loss in performance occasioned by the robustness, etc.). For this reason, it is beneficial to consider situations where a simpler approach can be employed. In this paper we consider two commonly encountered situations; the first consists of the detection of a time varying deterministic signal in additive (not necessarily Gaussian) noise, whereas the second generalizes this situation to include a time varying signal with random amplitude. Our results allow determining when the likelihood ratio is invariant to perturbations in the underlying distributions from their nominal values, thus admitting the employment of the

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nominal distributions to design the data processor.

II. DEVELOPMENT

The situation under consideration can be modeled as a choice between

$$H_0: \mu_0, \quad H_1: \mu_1$$

where μ_0 and μ_1 are finite measures defined on the Borel sets of $\mathbb{R}^n$. Let λ be a σ-finite dominating measure for μ_0 and μ_1, e.g. we could take $\lambda = \frac{1}{2}(\mu_0 + \mu_1)$. Consider first the detection of a time varying deterministic signal; that is, for a fixed element θ of $\mathbb{R}^n$ we set

$$\mu_1(B) = \mu_0(B-\{\theta\})$$

for all Borel sets B of $\mathbb{R}^n$. Note that the time varying signal θ is represented as a point in $\mathbb{R}^n$. We define the real, nonnegative, Borel measurable functions $f_0 = \frac{d\mu_0}{d\lambda}$ and $f_1 = \frac{d\mu_1}{d\lambda}$, and note that $f_1(\cdot) = f_0(\cdot - \theta)$ a.e. with respect to λ. We also let $S = \{x \in \mathbb{R}^n : f_0(x) f_1(x) > 0\}$, and note that the likelihood ratio $\Lambda(\cdot)$ given by

$$\Lambda(x) = \frac{f_1(x)}{f_0(x)}$$

exists and is positive for all $x \in S$. Note that S contains those elements of $\mathbb{R}^n$ for which we obtain a nondegenerate test; the cases where $\Lambda(x) = 0$ or $\Lambda(x) = \infty$ are easily checked.

Let $\mathcal{G}$ denote the class of all real, nonnegative, Borel measurable functions defined on $\mathbb{R}^n$; we will call an element of $\mathcal{G}$ a density. We then note that a wide class $\mathcal{C}$ of variations in the nominal density $f_0(\cdot)$ is generated by considering all densities $f: \mathbb{R}^n \to \mathbb{R}$ such that $f(\cdot) = (1-\varepsilon) f_0(\cdot) + \varepsilon g(\cdot)$, where $g \in \mathcal{G}$ and ε is a real number satisfying $0 \leq \varepsilon < 1$ (cf. the ε-contamination class of [1]). In this case the actual density under H_0 is not necessarily the nominal density $f_0(\cdot)$ but rather an arbitrary element of $\mathcal{C}$, with an associated $g \in \mathcal{G}$ and $\varepsilon \in [0,1)$. In the following we consider only those g belonging to $\mathcal{G}$. We thus are actually testing between

$$H_0: (1-\varepsilon)\, \mu_0 + \varepsilon \Gamma_0,$$

$$H_1: (1-\varepsilon)\, \mu_1 + \varepsilon \Gamma_1,$$

where for each Borel set B in $\mathbb{R}^n$,

$$\Gamma_0(B) = \int_B g\, d\lambda \quad \text{and} \quad \Gamma_1(B) = \Gamma_0(B-\{\theta\}).$$

Equivalently, we test between the densities

$$H_0: (1-\varepsilon)\, f_0(\cdot) + \varepsilon g(\cdot)$$

$$H_1: (1-\varepsilon)\, f_0(\cdot - \theta) + \varepsilon g(\cdot - \theta).$$

We are interested in the relevant likelihood ratio $\Lambda_{g,\varepsilon}(\cdot)$ given by, for $x\in S$,

$$\Lambda_{g,\varepsilon}(x) \triangleq \frac{f_0(x-\theta)+\varepsilon[g(x-\theta)-f_0(x-\theta)]}{f_0(x)+\varepsilon[g(x)-f_0(x)]}.$$

Note that $\Lambda_{g,\varepsilon}(x)$ exists for all $x\in S$ via our restrictions on ε. Note also that, for $x\in S$,

$$\frac{\partial\Lambda_{g,\varepsilon}(x)}{\partial\varepsilon} = \frac{[f_0(x)+\varepsilon(g(x)-f_0(x))][g(x-\theta)-f_0(x-\theta)]}{[f_0(x)+\varepsilon(g(x)-f_0(x))]^2}$$
$$- \frac{[f_0(x-\theta)+\varepsilon(g(x-\theta)-f_0(x-\theta))][g(x)-f_0(x)]}{[f_0(x)+\varepsilon(g(x)-f_0(x))]^2}.$$

We therefore have, for $x\in S$,

$$\frac{\partial\Lambda_{g,\varepsilon}(x)}{\partial\varepsilon} = 0 \quad \text{if and only if} \quad \frac{g(x-\theta)}{f_0(x-\theta)} = \frac{g(x)}{f_0(x)}.$$

This is true if and only if there exists a Borel measurable function $p: \mathbb{R}^n \to \mathbb{R}$ such that for all $x\in S$ we have $g(x)=p(x)f_0(x)$ and $p(x)=p(x-\theta)$. Since the above equivalence is true for all $\varepsilon\in[0,1)$, we have established the following result:

Theorem 1: Let A be a nonempty open subset of $[0,1)$ under the induced topology, and let $g\in\mathscr{G}$. We then have $\Lambda_{g,\varepsilon}(x) = \Lambda(x)$ for all $x\in S$ and for all $\varepsilon\in A$ if and only if there exists a Borel measurable function $p: \mathbb{R}^n \to \mathbb{R}$ such that for all $x\in S$,

$g(x) = p(x)f_0(x)$ and $p(x) = p(x-\theta)$.

Note that the above result characterizes the subclass of contaminants $g(\cdot)$ for which the likelihood ratio $\Lambda_{g,\varepsilon}(x)$ is invariant for each $x\in S$ as ε ranges over nonempty subsets which are open in $[0,1)$. This is true regardless of the nonempty open set A and in particular we can simply let ε be any number in $[0,1)$. The data processor of the detector could then be designed based on the nominal density with no degradation in performance for such contaminants g.

More generally, let us consider the extended situation where H_1 is nominally governed by the (Borel measurable) density $\tilde{f}_1(\cdot)$ given by

$$\tilde{f}_1(x) = \int_{\mathbb{R}^n} f_0(x-t)d\tilde{\mu}(t),$$

where for a fixed subset C of the nonzero integers the finite measure $\tilde{\mu}$ is defined on the Borel sets of $\mathbb{R}^n$ and is purely atomic with atoms $\{\{c\theta\}:c\in C\}$. Let $\tilde{S} = \{x\in\mathbb{R}^n: f_0(x)\tilde{f}_1(x)>0\}$. This case includes the previous one and extends it to the situation where the signal is allowed to possess a certain form of random amplitude. As before, we consider variations in the nominal density $f_0(\cdot)$ which lie in $\mathscr{G}$. In particular, let $g\in\mathscr{G}$ be such that there exists a Borel measurable function $p: \mathbb{R}^n\to\mathbb{R}$ with $g(x) = p(x)f_0(x)$ and

$p(x) = p(x-\theta)$ for $x \in \tilde{S}$. We then note that the likelihood ratio which corresponds to the nominal density $f_0(\cdot)$ is given on $\tilde{S}$ by

$$\tilde{\Lambda}(x) = \frac{\tilde{f}_1(x)}{f_0(x)},$$

whereas the one which corresponds to the variation $\tilde{f}(\cdot) = (1-\varepsilon)f_0(\cdot) + \varepsilon g(\cdot)$ is given, for all $x \in \tilde{S}$ and for all $\varepsilon \in [0,1)$, by

$$\tilde{\Lambda}_{g,\varepsilon}(x) = \frac{\tilde{f}_1(x) + \varepsilon \int_{\mathbb{R}^n} [g(x-t) - f_0(x-t)]d\tilde{\mu}(t)}{f_0(x) + \varepsilon[g(x) - f_0(x)]}$$

$$= \sum_{c \in C} \frac{f_0(x-c\theta) + \varepsilon[g(x-c\theta) - f_0(x-c\theta)]}{f_0(x) + \varepsilon[g(x) - f_0(x)]} \cdot k_c$$

for some countable collection of real constants k_c which depend only on c, θ, and $\tilde{\mu}$. Note that it then follows from Theorem 1 that for each $x \in \tilde{S}$, $\tilde{\Lambda}_{g,\varepsilon}(x) = \tilde{\Lambda}(x)$ for all $\varepsilon \in [0,1)$. We have therefore established the following result:

<u>Theorem 2</u>: Suppose $g \in \mathscr{G}$ and that there exists a Borel measurable function $p: \mathbb{R}^n \to \mathbb{R}$ with $g(x) = p(x)f_0(x)$ and $p(x) = p(x-\theta)$ for all $x \in \tilde{S}$. It then follows that

$\tilde{\Lambda}_{g,\varepsilon}(x) = \tilde{\Lambda}(x)$ for all $x \in \tilde{S}$ and for all $\varepsilon \in [0,1)$.

The above result is thus an extension of Theorem 1 to a more general situation; however, we retain only the sufficiency condition. This, of course, is the condition we often might want to employ in practice.

Note that a key element in both Theorem 1 and Theorem 2 is that a contaminant which is a periodic multiple (with period equal to the signal) of the nominal density leads to an unperturbed likelihood ratio. The results of Theorem 1 and Theorem 2 provide us with an easily used tool to construct remarkedly unintuitive examples of noise contaminants, all of which employ the same optimal data processor for a wide variety of fidelity criteria. For example, let $n = 1$ and $\theta = 0.1$. If

$$f_0(x) = \frac{1}{2} I_{[-1,1]}(x) = \begin{cases} \frac{1}{2} & \text{if } x \in [-1,1] \\ 0 & \text{otherwise} \end{cases},$$

i.e. the associated random variable is uniform on [-1,1], and if the perturbation of f_0 is given by

$$f(x) = \begin{cases} 4.5\times 10^7 + 5\times 10^{-2} & \text{if } x \in [\frac{i}{10} - \frac{10^{-9}}{2}, \frac{i}{10} + \frac{10^{-9}}{2}] \cap [-1,1] \\ & \quad \text{for } i = -10,-9,\ldots,10 \\ 0 & \text{if } x \in (-\infty,-1) \cup (1,\infty) \\ 5\times 10^{-2} & \text{otherwise} \end{cases},$$

then the resultant likelihood ratios are the same over $S = [-0.9,1]$, even though the two densities are very dissimilar. Notice in this example that $f(x)$ is nowhere equal to $f_0(x)$ on the interval [-1,1].

For another example, let $n = 1$, $\theta = 2$, and

$$f_0(x) = 0.05\ I_{[-10,10]}(x) = \begin{cases} 0.05 & \text{if } x\epsilon[-10,10] \\ 0 & \text{otherwise} \end{cases} .$$

Let D be a Cantor set (see, for example, [7]) in [0,1] with Lebesgue measure $\lambda\epsilon(0,1)$. If the perturbation of f_0 is given by

$$f(x) = \begin{cases} \frac{1}{20\lambda} & \text{if } x\epsilon D + \{2i\} \text{ for } i=-5,-4,\ldots,4 \\ 0 & \text{if } x\epsilon(-\infty,-10)\cup(10,\infty) , \\ \frac{1}{40-20\lambda} & \text{otherwise} \end{cases}$$

then the resultant likelihood ratios are the same over $S = [-8,10]$, even though the Cantor set results in a perturbation of f which is obviously very dissimilar to f_0. Notice that the maximum of the perturbation f can be made to exceed any preassigned real number.

III. CONCLUSION

We have considered the effect induced on the data processor of a signal detection system when the underlying distribution functions are varied about their nominal values. Having noted the crucial role played by the likelihood ratio in the detection system, we have therefore focused attention on the variation in the likelihood ratio itself. When testing first for the presence of a time varying deterministic signal, and then in a more general situation, we showed that the likelihood ratio is invariant over a class of perturbations of the nominal noise distribution.

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